Code : 031510

B.Tech 5th Semester Exam., 2019

SIGNALS AND SYSTEMS

Time : 3 hours *Full Marks* : 70

Instructions :

(i) *The marks are indicated in the right-hand margin.*

(ii) *There are* **NINE** *questions in this paper.*

(iii) *Attempt* **FIVE** *questions in all.*

(iv) *Question No.* **1** *is compulsory.*

1. Answer any *seven* of the following questions : 2×7=14

(a) Find the fundamental period and frequency of the signal

$$x(t) = \cos 18\pi t + \sin 12\pi t$$

(b) With an example, prove that cascade combination of an LTI system and its inverse system results an identity system.

(c) Check whether the system $y(t) = tx(t)$ is causal and stable.

(d) What is the physical significance of convolution?

(e) What do you mean by convergence of Fourier series?

(f) Prove that

$$x_{\text{even}}(t) \overset{T}{\leftrightarrow} \text{Re}\{a_k\}$$

where $x(t)$ is real and $x(t) \overset{T}{\leftrightarrow} a_k$.

(g) Determine the Laplace transform for the signal $x(t) = e^{-5t}u(t-1)$.

(h) For an LTI system to be causal and stable, what should be the condition on ROC and locations of poles?

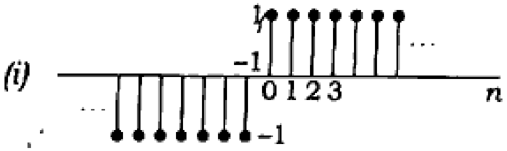
(i) Find the *z*-transform and ROC of

$$x[n] = \left(\frac{1}{5}\right)^n u(n-3)$$

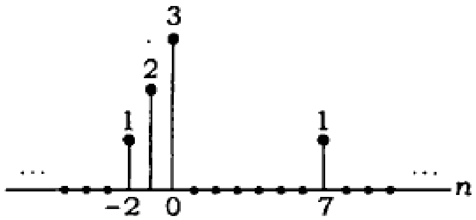
(j) State and prove time reversal property of *z*-transform.

2. *(a)* Compute and plot the even and odd parts of the following signals : 8

(i)



(ii)



(b) Let $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t}u(t)$. Compute : 6

(i) $y(t) = x(t) * h(t)$

(ii) $g(t) = (dx(t)/dt) * h(t)$

3. *(a)* For each of the following systems, check the properties of linearity, time-invariance causality and BIBO stability : 8

(i) $y(t) = x(t/2)$

(ii) $y[n] = nx[n]$

(b) Compute and plot the convolution

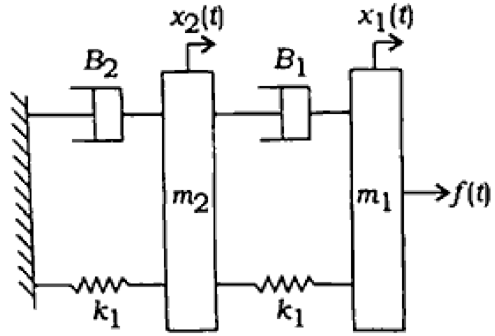
$$y[n] = x[n] * h[n]$$

where

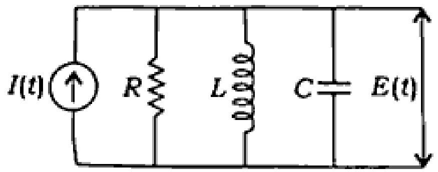
$x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$ and $h[n] = u[n+2]$ 6

4. *(a)* Obtain the differential equations for the following systems : 10

(i)



(ii)



(b) Explain the 'property of sifting' of discrete-time unit impulse. 4

5. *(a)* State and prove the following properties of continuous-time Fourier series : 10

(i) Frequency shifting

(ii) Periodic convolution in time-domain

(iii) Time scaling

(iv) Differentiation in time-domain

(b) Calculate the Fourier series coefficients for the periodic signal

$$x(t)=\begin{cases}2\ , & 0\le t<2\\ -2\ , & 2\le t<4\end{cases}$$

with fundamental frequency $\omega_0=\pi/2$. 4

6. (a) State and prove the following properties of discrete-time Fourier transform : 10

(i) Time shifting

(ii) Time expansion

(iii) Multiplication of two signals in time-domain

(iv) Differentiation in frequency

(b) Compute the Fourier transform of

$$x[n]=\left(\frac{1}{4}\right)^{|n-1|}$$ 4

7. (a) Consider an LTI system whose response to the input $x(t)=(e^{-t}+e^{-3t})u(t)$ is $y(t)=(2e^{-t}-2e^{-4t})u(t)$. Using Laplace transforms, find the impulse response of the system. Also compute its frequency response. 10

(b) Determine the inverse Laplace transform of

$$X(s)=\frac{(s+1)}{(s+1)^2+4},\ \text{Re}\{s\}>-1$$ 4

8. (a) Determine the impulse response of the system described by the difference equation

$$y[n]=\frac{1}{2}[x[n]+x[n-1]+y[n-1]]$$

Assume that the system is initially relaxed. 7

(b) Solve the following linear difference equation :

$$y[n]+\frac{1}{2}y[n-1]-\frac{1}{4}y[n-2]=0$$

Given that $y[-1]=y[-2]=1$. 7

9. (a) Determine all possible signals of $x(n)$ associated with the following z-transforms : 10

(i) $X(z)=\dfrac{1}{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}}$

(ii) $X(z) = \dfrac{5}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}$

(b) Obtain the z-transform and ROC of the following sequence : 4

$$x(n) = \left(\frac{1}{2}\right)^n [u[n] - u[n-10]]$$

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